A quantum walk simulation of extra dimensions with warped geometry

Andreu Anglés-Castillo[1] and Armando Pérez[1]

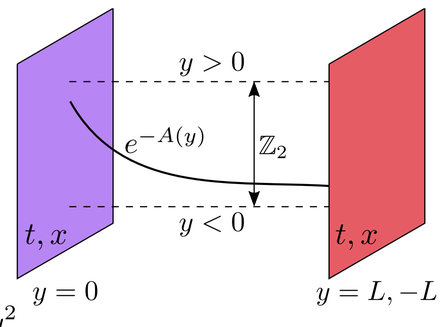
[1]*Universitat de València-CSIC, Departament de Física Teòrica & IFIC, Burjassot (València), 46100, Spain*

The Randall-Sundrum model

The Randall-Sundrum (RS) was originally proposed to address the *hierarchy problem.* It assumes the existence of an extra dimension that extends between two branes. The Standard Model particles are assumed to be confined in one of these branes, while gravity spans over the whole space.

Orbifold

$$S^1 : \ y \sim y + L \ ,$$
$$\mathbb{Z}_2 : \ y \sim -y$$



Metric

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

Quantum Walk of spinors

We employ a Quantum Walk (QW) in order to simulate the evolution of spinors that move within this geometry. We consider a simplied version with 1 ordinary dimension and 1 extra dimension (plus time). The evolution of spinors is given by the Dirac equation in curved space-time, which can be codified into a QW [1].

Dirac Equation

$$i\partial_t\chi = \mathcal{H}\chi, \quad B^x = \gamma^0\gamma^1 \ , \quad B^y = e^{-k|y|}\gamma^0\gamma^2$$
$$\mathcal{H} = -\frac{i}{2}\{B^x, \partial_x\} - \frac{i}{2}\{B^y, \partial_y\} + \gamma_0 e^{-k|y|}m$$

Boundary Conditions

$$\chi(x^\mu, y + 2L) = \chi(x^\mu, y)$$
$$\chi(x^\mu, -y) = \sigma_z\chi(x^\mu, y)$$

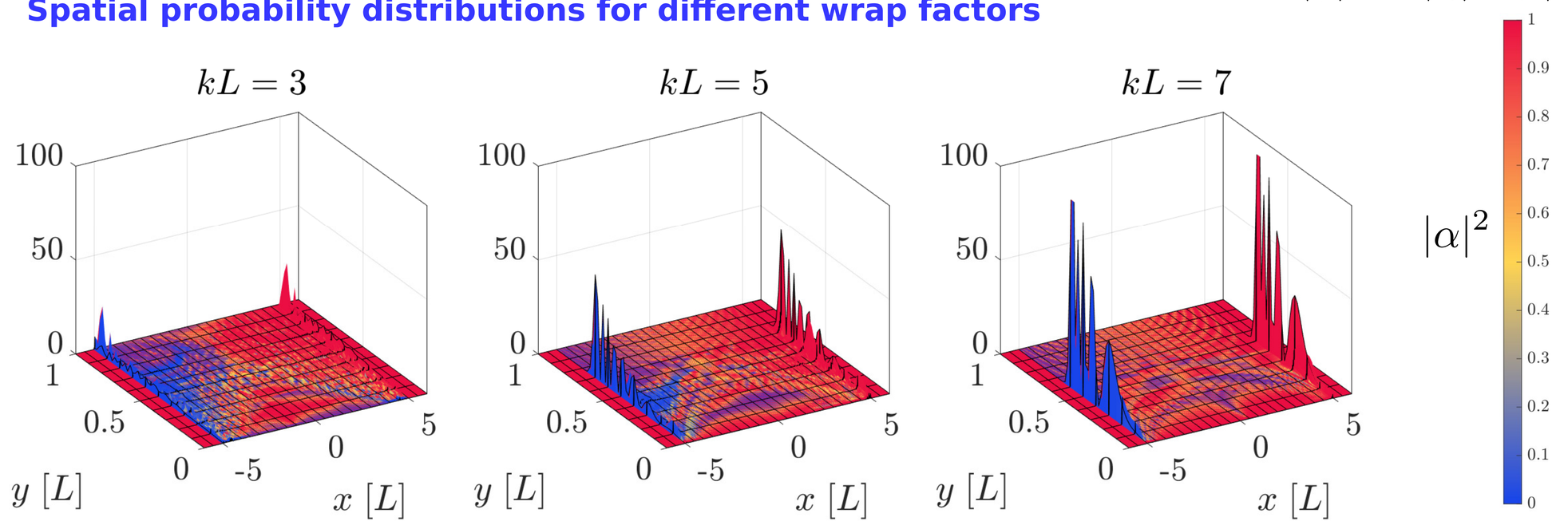
Quantum Walk

$$|\chi_{j+1}\rangle = U|\chi_j\rangle$$
$$\langle x = \epsilon r, y = \epsilon s|\chi_j\rangle = \begin{pmatrix}\chi^{\uparrow}_{j,r,s}\\ \chi^{\downarrow}_{j,r,s}\end{pmatrix}$$

[1] Arnault, P. & Debbasch, F. Quantum walks and gravitational waves. *Annals Phys.* 383, 645 – 661

Spatial probability distributions for different wrap factors

$$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$



Confinement

Expected value of the probability distribution along the extra dimension y, for different values of the warp coefficient.

Entanglement

Evolution of the entanglement entropy between he spatial degree o freedom and the coin of the walker for different values of the warp coefficient. The dotted line represent the minimum value the entropy can reach for very high values of the warp factor.

Conclusions

In this work we employed a QW implementation which is able to simulate the Dirac equation in a curved space-time corresponding to the RS model. The model predicts the confinement of fermions in one of the edges of the extra dimension. The QW was able to correctly reproduce the fenomenology of the original model. We also studied the entanglement entropy that exists between the position and the coin of the walker as a function of the warp coefficient. These results constitute an example of the abiliy of QWs to simulate relevant high energy theories in physics.